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Candidate surname		Other names	
Centre Number	Candidate Number		
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**Pearson Edexcel Level 3 GCE**

**Tuesday 6 June 2023**

Afternoon (Time: 2 hours)	Paper reference	<b>9MA0/01</b>
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**Mathematics**

**Advanced**

**PAPER 1: Pure Mathematics 1**

<p><b>You must have:</b></p> <p>Mathematical Formulae and Statistical Tables (Green), calculator</p>	<p>Total Marks</p>
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**Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.**

### Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided  
– *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

### Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 15 questions in this question paper. The total mark for this paper is 100.
- The marks for **each** question are shown in brackets  
– *use this as a guide as to how much time to spend on each question.*

### Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ►

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1. Find

$$\int \frac{x^{\frac{1}{2}}(2x-5)}{3} dx$$

writing each term in simplest form.

(4)

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Question 1 continued

(Total for Question 1 is 4 marks)

2.

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

$$f(x) = 4x^3 + 5x^2 - 10x + 4a \quad x \in \mathbb{R}$$

where  $a$  is a positive constant.

Given  $(x - a)$  is a factor of  $f(x)$ ,

(a) show that

$$a(4a^2 + 5a - 6) = 0 \quad (2)$$

(b) Hence

(i) find the value of  $a$

(ii) use algebra to find the exact solutions of the equation

$$f(x) = 3 \quad (4)$$

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Question 2 continued

(Total for Question 2 is 6 marks)

3. Relative to a fixed origin  $O$

- the point  $A$  has position vector  $5\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}$
- the point  $B$  has position vector  $2\mathbf{i} + 4\mathbf{j} + a\mathbf{k}$

where  $a$  is a positive integer.

(a) Show that  $|\vec{OA}| = \sqrt{38}$

(1)

(b) Find the smallest value of  $a$  for which

$$|\vec{OB}| > |\vec{OA}|$$

(2)

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Question 3 continued

(Total for Question 3 is 3 marks)

4.

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

The curve  $C$  has equation  $y = f(x)$  where  $x \in \mathbb{R}$

Given that

- $f'(x) = 2x + \frac{1}{2} \cos x$
  - the curve has a stationary point with  $x$  coordinate  $\alpha$
  - $\alpha$  is small
- (a) use the small angle approximation for  $\cos x$  to estimate the value of  $\alpha$  to 3 decimal places.

(3)

The point  $P(0, 3)$  lies on  $C$

- (b) Find the equation of the tangent to the curve at  $P$ , giving your answer in the form  $y = mx + c$ , where  $m$  and  $c$  are constants to be found.

(2)

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Question 4 continued

(Total for Question 4 is 5 marks)

5. A continuous curve has equation  $y = f(x)$ .

The table shows corresponding values of  $x$  and  $y$  for this curve, where  $a$  and  $b$  are constants.

$x$	3	3.2	3.4	3.6	3.8	4
$y$	$a$	16.8	$b$	20.2	18.7	13.5

The trapezium rule is used, with all the  $y$  values in the table, to find an approximate area under the curve between  $x = 3$  and  $x = 4$

Given that this area is 17.59

- (a) show that  $a + 2b = 51$

(3)

Given also that the sum of all the  $y$  values in the table is 97.2

- (b) find the value of  $a$  and the value of  $b$

(3)



Question 5 continued

(Total for Question 5 is 6 marks)

6.

$$a = \log_2 x$$

$$b = \log_2(x + 8)$$

Express in terms of  $a$  and/or  $b$ 

(a)  $\log_2 \sqrt{x}$

(1)

(b)  $\log_2(x^2 + 8x)$

(2)

(c)  $\log_2\left(8 + \frac{64}{x}\right)$

Give your answer in simplest form.

(3)

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Question 6 continued

(Total for Question 6 is 6 marks)

7. The function  $f$  is defined by

$$f(x) = 3 + \sqrt{x - 2} \quad x \in \mathbb{R} \quad x > 2$$

(a) State the range of  $f$

(1)

(b) Find  $f^{-1}$

(3)

The function  $g$  is defined by

$$g(x) = \frac{15}{x - 3} \quad x \in \mathbb{R} \quad x \neq 3$$

(c) Find  $gf(6)$

(2)

(d) Find the exact value of the constant  $a$  for which

$$f(a^2 + 2) = g(a)$$

(2)

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Question 7 continued



Question 7 continued

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Question 7 continued

(Total for Question 7 is 8 marks)

8.

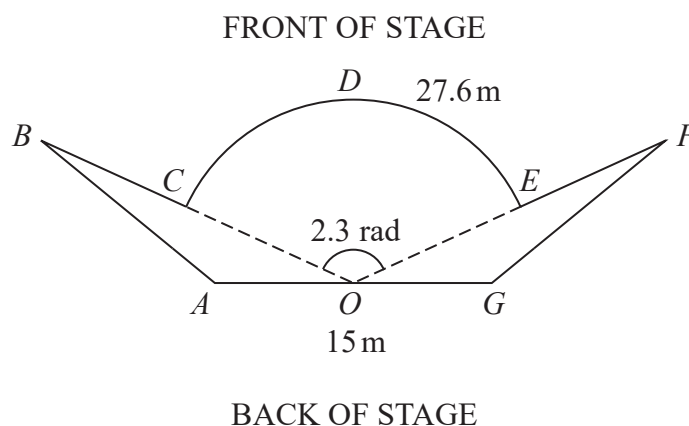
Diagram **NOT**  
accurately drawn

Figure 1

Figure 1 shows the plan view of a stage.

The plan view shows two congruent triangles  $ABO$  and  $GFO$  joined to a sector  $OCDEO$  of a circle, centre  $O$ , where

- angle  $COE = 2.3$  radians
- arc length  $CDE = 27.6$  m
- $AOG$  is a straight line of length 15 m

(a) Show that  $OC = 12$  m.

(2)

(b) Show that the size of angle  $AOB$  is 0.421 radians correct to 3 decimal places.

(2)

Given that the total length of the front of the stage,  $BCDEF$ , is 35 m,

(c) find the total area of the stage, giving your answer to the nearest square metre.

(6)

Question 8 continued



Question 8 continued

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Question 8 continued

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(Total for Question 8 is 10 marks)



9. The first three terms of a geometric sequence are

$$3k + 4 \quad 12 - 3k \quad k + 16$$

where  $k$  is a constant.

(a) Show that  $k$  satisfies the equation

$$3k^2 - 62k + 40 = 0 \quad (2)$$

Given that the sequence converges,

(b) (i) find the value of  $k$ , giving a reason for your answer,

(ii) find the value of  $S_{\infty}$  (5)

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Question 9 continued

(Total for Question 9 is 7 marks)

10. A circle  $C$  has equation

$$x^2 + y^2 + 6kx - 2ky + 7 = 0$$

where  $k$  is a constant.

(a) Find in terms of  $k$ ,

- (i) the coordinates of the centre of  $C$
- (ii) the radius of  $C$

(3)

The line with equation  $y = 2x - 1$  intersects  $C$  at 2 distinct points.

(b) Find the range of possible values of  $k$ .

(6)

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Question 10 continued



Question 10 continued

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Question 10 continued

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(Total for Question 10 is 9 marks)



11.

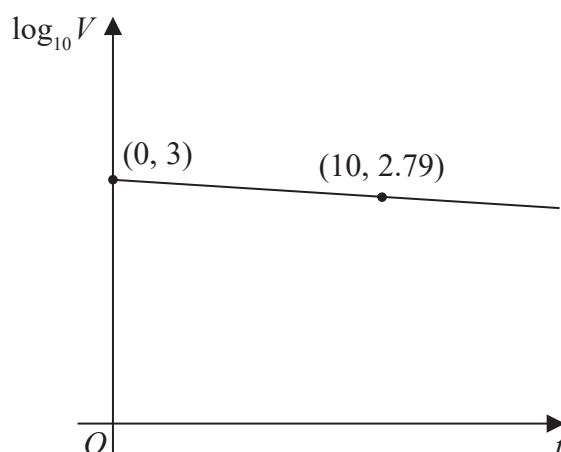


Figure 2

The value,  $V$  pounds, of a mobile phone,  $t$  months after it was bought, is modelled by

$$V = ab^t$$

where  $a$  and  $b$  are constants.

Figure 2 shows the linear relationship between  $\log_{10} V$  and  $t$ .

The line passes through the points  $(0, 3)$  and  $(10, 2.79)$  as shown.

Using these points,

- (a) find the initial value of the phone, (2)

- (b) find a complete equation for  $V$  in terms of  $t$ , giving the exact value of  $a$  and giving the value of  $b$  to 3 significant figures. (3)

Exactly 2 years after it was bought, the value of the phone was £320

- (c) Use this information to evaluate the reliability of the model. (2)

Question 11 continued



Question 11 continued

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Question 11 continued

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(Total for Question 11 is 7 marks)



12.

$$y = \sin x$$

where  $x$  is measured in radians.

Use differentiation from first principles to show that

$$\frac{dy}{dx} = \cos x$$

You may

- use without proof the formula for  $\sin(A \pm B)$
- assume that as  $h \rightarrow 0$ ,  $\frac{\sin h}{h} \rightarrow 1$  and  $\frac{\cos h - 1}{h} \rightarrow 0$

(5)

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Question 12 continued

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(Total for Question 12 is 5 marks)



13. On a roller coaster ride, passengers travel in carriages around a track.

On the ride, carriages complete multiple circuits of the track such that

- the maximum vertical height of a carriage above the ground is 60 m
- a carriage starts a circuit at a vertical height of 2 m above the ground
- the ground is horizontal

The vertical height,  $H$  m, of a carriage above the ground,  $t$  seconds after the carriage starts the first circuit, is modelled by the equation

$$H = a - b(t - 20)^2$$

where  $a$  and  $b$  are positive constants.

(a) Find a complete equation for the model.

(3)

(b) Use the model to determine the height of the carriage above the ground when  $t = 40$

(1)

In an alternative model, the vertical height,  $H$  m, of a carriage above the ground,  $t$  seconds after the carriage starts the first circuit, is given by

$$H = 29 \cos(9t + \alpha)^\circ + \beta \quad 0 \leq \alpha < 360^\circ$$

where  $\alpha$  and  $\beta$  are constants.

(c) Find a complete equation for the alternative model.

(2)

Given that the carriage moves continuously for 2 minutes,

(d) give a reason why the alternative model would be more appropriate.

(1)



Question 13 continued



Question 13 continued

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Question 13 continued

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(Total for Question 13 is 7 marks)



14. Prove, using algebra, that

$$(n + 1)^3 - n^3$$

is odd for all  $n \in \mathbb{N}$

(4)

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Question 14 continued

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(Total for Question 14 is 4 marks)



15. A curve has equation  $y = f(x)$ , where

$$f(x) = \frac{7xe^x}{\sqrt{e^{3x} - 2}} \quad x > \ln \sqrt[3]{2}$$

(a) Show that

$$f'(x) = \frac{7e^x(e^{3x}(2-x) + Ax + B)}{2(e^{3x} - 2)^{\frac{3}{2}}}$$

where  $A$  and  $B$  are constants to be found.

(5)

(b) Hence show that the  $x$  coordinates of the turning points of the curve are solutions of the equation

$$x = \frac{2e^{3x} - 4}{e^{3x} + 4}$$

(2)

The equation  $x = \frac{2e^{3x} - 4}{e^{3x} + 4}$  has two positive roots  $\alpha$  and  $\beta$  where  $\beta > \alpha$

A student uses the iteration formula

$$x_{n+1} = \frac{2e^{3x_n} - 4}{e^{3x_n} + 4}$$

in an attempt to find approximations for  $\alpha$  and  $\beta$

Diagram 1 shows a plot of part of the curve with equation  $y = \frac{2e^{3x} - 4}{e^{3x} + 4}$  and part of the line with equation  $y = x$

Using Diagram 1 on page 42

(c) draw a staircase diagram to show that the iteration formula starting with  $x_1 = 1$  can be used to find an approximation for  $\beta$

(1)

Use the iteration formula with  $x_1 = 1$ , to find, to 3 decimal places,

(d) (i) the value of  $x_2$

(ii) the value of  $\beta$

(3)

Using a suitable interval and a suitable function that should be stated

(e) show that  $\alpha = 0.432$  to 3 decimal places.

(2)





Question 15 continued



### Question 15 continued

Only use the copy of Diagram 1 if you need to redraw your answer to part (c).

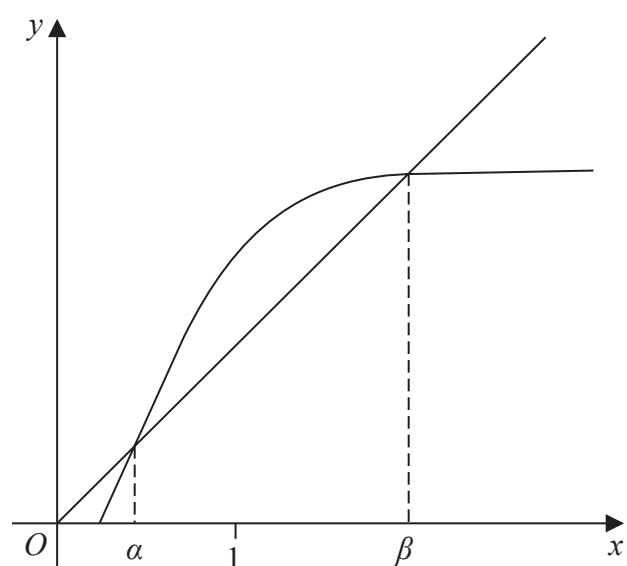
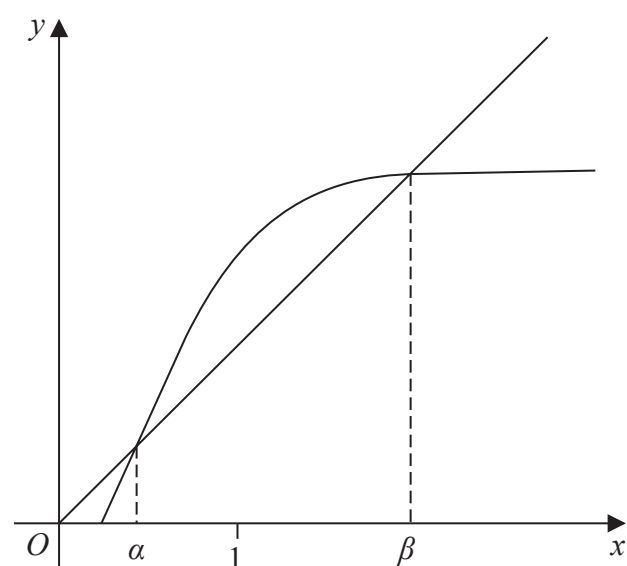


Diagram 1



copy of Diagram 1

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Question 15 continued



Question 15 continued

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(Total for Question 15 is 13 marks)

**TOTAL FOR PAPER IS 100 MARKS**

